§3.3 5d $\mathcal{N}=1$ SCFT's

8 supercharg
R-symmetry: $SU(2)_R$
mass-less representations:

- hypermultiplet (4 real scalars, 1 spinor)
- vector multiplet (vector, real scalar, spinor)

duality: vector rep. is dual to tensor rep.: $A_\mu \Longleftrightarrow B_{\mu\nu}$

notation: vector multiplet $= (\phi, A_\mu, \lambda)$

Moduli spaces of vacua:

- Coulomb branch:
  parametrized by expectation values $\langle \phi^i \rangle$ for scalars in vector multiplets.

- Higgs branch:
  parametrized by expectation values of scalars $(\varphi_1, \varphi_2, \varphi_3, \varphi_4)$ in Hypermultiplets
  $\longrightarrow$ HyperKähler manifolds

Lagrangian descriptions:

- Kinetic terms:

$$\mathcal{L}_{kin} = \left(-\frac{1}{4} F^i_{\mu\nu} F^{\mu\nu j} - \frac{1}{2} \bar{\lambda}^i \not{\partial} \lambda^j - \frac{1}{2} \partial_\mu \phi^i \partial^\mu \phi^j\right) a_{ij}(\phi)$$

where $a_{ij}(\phi) = \frac{\partial}{\partial \phi^i} \frac{\partial}{\partial \phi^j} \mathcal{F}(\phi)$

and $\mathcal{F}(\phi)$ is the so called "pre-potential":

$$\mathcal{F} = c_0 + c_i \phi^i + c_{ij} \phi^i \phi^j + c_{ijk} \phi^i \phi^j \phi^k$$

The constants c_0 and c_i do not affect the Lagrangian and can be set to zero.

- Chern-Simons terms:

$$\mathcal{L}_{CS} = \left(-\frac{1}{24} \varepsilon^{\mu\nu\lambda\rho\sigma} A^i_\mu F^j_{\nu\lambda} F^k_{\rho\sigma} + \text{fermions}\right) \mathcal{F}_{ijk}(\phi)$$

where $\mathcal{F}_{ijk} = \frac{\partial}{\partial \phi^i \partial \phi^j \phi^k} \mathcal{F}(\phi)$

Consider the case of one vector mult.:

$$F = \frac{1}{2g^2} \phi^2 + \frac{c}{6} \phi^3$$

with g and c real constants.

One gets kinetic terms depending on gauge coupling g and Chern-Simons term

$$c\left(\phi (\partial\phi)^2 + \phi F_{\mu\nu}^2 + A \wedge F \wedge F + \cdots\right)$$

Perturbative dynamics:

Consider $U(1)$ gauge theories with N_f "electron" hypermultiplets of charge one and $SU(2)$ gauge theories with N_f "quark" hypermultiplets.

→ not renormalizable (field theories with cutoff)

→ c can be only generated at one-loop (only chiral multiplets contribute, contribution of hyper-mult. = − vector-mult.)

- For $U(1)$ theory:

$$c = -a N_f$$

- For $SU(2)$:

$$c = a(8 - N_f)$$

→ set $c = \begin{cases} -N_f & \text{for } U(1) \\ 2(8-N_f) & \text{for } SU(2) \end{cases}$

- singularities:

at $\phi = 0$ (electron becomes mass-less)
extend beyond by using symmetry
$\phi \to -\phi$

$$\to c\left(|\phi|(\partial\phi)^2 + |\phi| F_{\mu\nu}^2 + \varepsilon(\phi)\, A \wedge F \wedge F + \dots\right)$$

Effective gauge coupling in $U(1)$ theory:

$$\frac{1}{g_{eff}^2} = \frac{1}{g^2} + c|\phi|$$

$\to$ divergence at $\phi = \pm \frac{1}{cg^2}$

$\to$ UV theory is not well-defined!

- For $SU(2)$ theory moduli space is modded out by $\phi \to -\phi$ and we take $\phi \geq 0$ with singularity at $\phi = 0$

  $\to$ effective gauge coupling:

$$\frac{1}{g_{eff}^2} = \frac{1}{g^2} + 16\phi - \sum_i |\phi - m_i| - \sum_i |\phi + m_i|$$

  $\to$ no singularities for $N_f \leq 8$

  (for $N_f > 8$ high energy theory is not well-defined)

<u>$N_f < 8$:</u>

Consider strong coupling limit $g = \infty$

$\to$ SCFT fixed point

From the point of view of the SCFT, $\frac{1}{g_{eff}^2} \neq 0$ turns on a relevant deformation as it is dimensionfull $\Delta\left(\frac{1}{g^2}\right) = 1$

Equivalently, $\Delta(F_{\mu\nu}^2) = 4 < 5 \to$ relevant def.

We classified relevant deformations of 5d $\mathcal{N}=1$ SCFT's in § 2.2

$\rightarrow$ only rel. def. : Flavour current !

In our case: $j = *(F \wedge F)$

$\partial_\mu j^\mu = 0 \rightarrow$ global $U(1)_I$ symmetry

I stands for instanton

$\int d^4x \, j^0$ = instanton number in 4d

Take $\frac{1}{g^2} \sim m_0$ to reside in a background vector superfield (scalar component)

Corresponding BPS objects with

$$Z = m_0 \int d^4x \, j^0$$

are instanton particles

Total central charge

$$Z_{total} = \underbrace{I_3}_{\text{electric charge}} \phi + I m_0$$

The mixing of the two symmetries gives $m_0 \sim \frac{1}{g^2} \rightarrow \frac{1}{g^2_{eff}}$ and

$$Z_{total} = (I_3 + cI)\phi + I m_0$$

Strong coupling fixed points

Seiberg argues that the global symmetry at the strong coupling fixed points is

$$E_{N_f+1} \quad \text{for } N_f \leq 8$$

where $E_5 = Spin(10)$, $E_4 = SU(5)$, $E_3 = SU(3) \times SU(2)$,

$$E_2 = SU(2) \times U(1) \text{ and } E_1 = SU(2)$$

while $E_{6,7,8}$ correspond to exceptional groups

The global symmetries of the IR theories are

$$SO(2N_f) \times U(1)_I \subset E_{N_f+1}$$

then maximal subalgebras.

Flavor deformations of the SCFT correspond to turning on background gauge fields in the Cartan subalgebra of E_n:

m_i $(i=0,\dots,n-1)$.

Turning on $m_0 \xrightarrow{\text{flows}}$ $SU(2)$ with $n-1$ quarks

Turning on m_i for $i=p,\dots,N_f$

$\xrightarrow{\text{flow}}$ E_p fixed point

$\xrightarrow{\text{turn on } m_0}$ $SU(2)$ with $p-1$ flavors.